Near Horizon Geometry equation, degenerate Killing horizons

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The Near Horizon Geometry (NHG) equation can be imposed on pairs (g, ω), a metric tensor g and a differential 1-form ω defined on an n-dimensional manifold S. It reads

$$\nabla_{(A}\omega_{B)} + \omega_A\omega_B - \frac{1}{2}R_{AB} + \frac{2}{n}\Lambda\, g_{AB} = 0, \qquad \Lambda \in \mathbb{R}, \tag{1}$$

where Λ is the cosmological constant in the applications to General Relativity. The NHG equation emerges as a constraint on the induced degenerate metric tensors and the second fundamental forms of $n+1$-dimensional extremal Killing horizons in vacuum $n+2$-dimensional spacetimes [1, 2]. Recently, the NHG equation has been intensively studied and new results have been achieved. Most of them are applicable to 4-dimensional spacetimes. The holomorphic meaning of the equation has been used to solve it completely for all the compact manifolds S of the Euler characteristics

$$\chi_E(S) \le 0 \tag{2}$$

(that is genus > 0) [3]. For topological 2-sphere S, all the axisymmetric solutions are known and recently it has been shown they are isolated in the space of all the solutions [4, 5]. However an existence of non-axisymmetric solutions in still a mistery, although an interesting integrability condition has been recently derived. A knowledge of all possible extremal Killing horizons would be important for filling gaps in the black hole uniqueness theorems. In fact, much less rigorous results are known about stationary extremal (that is degenerate) black holes than about the stationary non-extremal ones [6]. The 1-form ω encodes information about the covariant derivative D induced on extremal Killing horizons. It turns out, that there is one more constraint equation on g and D implied by the vacuum Einstein equations and by the extremal Killing horizon assumption [7].

The name NHG comes from the 1-1 correspondence between solutions to the equation and some class of solutions to vacuum Einstein's equations with the cosmological constant Λ that describe vicinity of extremal Killing horizons [8, 9]. They have become famous as background for the Kerr/CFT correspondence [10]. One more application of the equation is construction of Kund's class spacetimes [11] .

In our talk we will review the recent results on the NHG equation and present the new extremal Killing horizon constraint.

[1] A. Ashtekar, C. Beetle, J. Lewandowski, *Geometry of Generic Isolated Horizon*, Class. Quantum Grav. **19** (2002), 1195–1225, arXiv:gr-qc/0111067.

[2] J. Lewandowski, T. Pawłowski *Quasilocal rotating black holes in higher dimensions: geometry*, Class. Quantum Grav. **22** (2005), 1573.

[3] D. Dobkowski-Ryłko, W. Kamiński, J. Lewandowski, A. Szereszewski, *The Near Horizon Geometry equation on compact 2-manifolds including the general solution for g ¿ 0*, Phys.Lett. **B785** (2018) 381-385.

[4] J. Lewandowski, T. Pawłowski, *Extremal Isolated Horizons: A Local Uniqueness Theorem*, Class. Quantum Grav. **20** (2003), 587–606, arXiv:gr-qc/0208032.

[5] P.T. Chruściel, S.J. Szybka, P. Tod, *Towards a classification of vacuum near-horizons geometries*, Class. Quantum Grav. **35** (2017), 015002, arXiv:arXiv:1707.01118 [gr-qc].

[6] P.T. Chruściel, J. Lopes Costa, M. Heusler, *Stationary Black Holes: Uniqueness and Beyond*, Living Rev. Rel. **15** (2012), 7, arXiv:1205.6112 [gr- qc].

[7] J. Lewandowski, A. Szereszewski, *A new constraint for degenerate Killing horizons*, to be published (2019).

[8] T. Pawłowski, J. Lewandowski, J. Jezierski, *Spacetimes foliated by Killing horizons*, Class. Quantum Grav. **21** (2004), 1237–1252, arXiv:gr-qc/0306107.

[9] H. K. Kunduri, J. Lucietti, *Classification of Near-Horizon Geometries of Extremal Black Holes*, Living Rev. Rel. **16** (2013), 8, http://www.livingreviews.org/lrr-2013-8, arXiv:abs/1306.2517.

[10] H.S. Reall, *Higher dimensional black holes and supersymmetry*, Phys. Rev. D **68** (2003), 024024, arXiv:hep-th/0211290.

[11] J. Lewandowski, A. Szereszewski, P. Waluk *Spacetimes foliated by non-expanding and Killing horizons: higher dimension*, Phys. Rev. **D94** (2016), 064018.