Cosmological perturbations in the Regge-Wheeler formalism

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We study linear perturbations of the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model in the Regge-Wheeler formalism which is a standard framework to study perturbations of spherically-symmetric black holes. In particular, we show that the general solution of linear perturbation equations can be given in terms of two copies of a master scalar satisfying scalar wave equation on the FLRW background (with a Regge-Wheeler/Zerilli type potential) thus representing two gravitational degrees of freedom, and one scalar satisfying a transport type equation representing (conformal) matter perturbation [1]. We expect our formalism to be easily extended to include nonlinear perturbations, akin to to the recent work [2].

While the standard framework for studying cosmological perturbations is based on 1+3 splitting of metric perturbations resulting in scalar-vector-tensor ($\mathcal{S}-\mathcal{V}-\mathcal{T}$) sectors of perturbations, the standard framework for studying metric perturbations of Schwarzschild black holes – the Regge-Wheeler (RW) formalism [3] – is based on 2+2 splitting of metric perturbation resulting in polar and axial sectors of perturbations (after expansion into suitably chosen polar/axial spherical harmonics). The key result of Schwarzschild black hole perturbation theory is that at linear level the general perturbation can be given in terms of only two (axial/polar) master scalars satisfying scalar wave equation on the Schwarzschild background [4] with Regge-Wheeler [3] and Zerilli [5] potentials for axial and polar sectors respectively (more precisely, this holds for any multipole $\ell \geq 2$; the monopole $\ell = 0$ and dipole $\ell = 1$ cases need some special treatment). Recently, this result was generalized to nonlinear perturbations [2]. We show that, once the Regge-Wheeler formalism is applied, the same structure emerges also for FLRW perturbations. We use standard conformal coordinates in which the FLRW line element takes the form:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + dq^2 + f^2(q)d\Omega_2^2\right], \qquad (1)$$

where $f(q) = q, \sin q, \sinh q$ for the flat, closed, and open universes respectively, and $d\Omega_2^2$ is the line element on a unit sphere. Then the solution of Einstein equations linearized around FLRW solution (1) can be given (for each spherical multipole $\ell > 1$) in terms of a master scalar $\Phi_\ell(\tau, q)$ satisfying a scalar wave equation on the FLRW background with a potential:

$$\left(-\Box_{\bar{g}} + V_\ell(\tau, q)\right)\frac{\Phi_\ell(\tau, q)}{f(q)} = 0, \qquad (2)$$

where

$$V_\ell(\tau, q) = \frac{1}{a^2(\tau)}\left(\frac{\ell(\ell+1)}{f^2(q)} + 2\dot{\mathcal{H}}(\tau)\right) \qquad (3)$$

and $\mathcal{H} = \mathcal{H}(\tau) = \left(\partial_\tau a(\tau)\right)/a(\tau)$ is the *conformal* Hubble constant. This potential is an analogue of the Regge-Wheeler/Zerilli potentials [3, 5] in the case of Schwarzschild black hole perturbations. The details can be found in [1]

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