Proper time hypersurfaces and cosmic time

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Abstract

In cosmology, it is natural to define hypersurfaces of "cosmic time" based on the proper time of the matter worldlines. However Ellis (2014) warns such hypersurfaces may become timelike in places. While this would not affect any physical phenomena, it is a conceptually inelegant definition of simultaneity. Note it is well known that orthogonal proper time hypersurfaces exist if and only if there is zero vorticity and acceleration, however for more general worldlines the hypersurfaces will not be orthogonal.

I first examine some illustrative toy models. In Schwarzschild spacetime, the static congruence based on the usual timelike Killing vector field has both acceleration, and near the horizon the proper time hypersurfaces quickly turn timelike due to gravitational time-dilation. In Minkowski spacetime, the rigidly rotating disc has both vorticity and acceleration, hence even an initial hypersurface cannot be chosen as orthogonal, and the time-dilation near the rim quickly leads to timelike hypersurfaces. The effect is cumulative, as even for small accelerations, the desynchronisation grows over time.

In the general case, the hope is to "untwist" the vorticity at least for some initial hypersurface, then introduce a lapse (for cosmic time relative to proper time) to compensate for acceleration. I report on progress towards this goal. Some known concepts turn up, including the (non-)geodesic deviation equation, and acceleration potentials (Ehlers 1961). In conclusion, the idea is to modify proper time hypersurfaces to determine the most sensible cosmic time.

- Ehlers, J. 1961 (translated 1993). Contributions to the relativistic mechanics of continuous media
- Ellis, G. 2014. The evolving block universe and the meshing together of times, Ann N.Y. Acad. Sci.