Hopf-algebraic deformations of 3d spacetime symmetries

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Abstract

It is widely argued, especially in the phenomenological approach to quantum gravity, that at the Planckian scales the geometry of spacetime may become noncommutative. A closely related effect is the generalization of classical spacetime symmetries into quantum-deformed algebras and groups, which are expected to have the structure of Hopf algebras and be characterized by the (classical) r-matrices – solutions of the Yang-Baxter equations. The best known example is the κ-Poincaré algebra. Such quantum algebras and groups actually arise in the convincing way in 2+1 dimensions, where gravity can be described as a Chern-Simons theory with the gauge group given by the isometry group of spacetime, the latter depending on the metric signature and cosmological constant. For non-vanishing cosmological constant the relevant algebras are $\mathfrak{o}(4)$, $\mathfrak{o}(3,1)$ and $\mathfrak{o}(2,2)$. They can also be treated as real forms of the complex algebra $\mathfrak{o}(4;\mathbb{C})$, which has recently been applied [1] to classify all of their possible quantum deformations. We have further expanded this line of inquiry [2] by studying the quantum İnönü-Wigner contractions, i.e. calculating the limit of vanishing cosmological constant of deformations of the $\mathfrak{o}(4)$, $\mathfrak{o}(3,1)$ and $\mathfrak{o}(2,2)$ algebras, which leads to either deformed 3d inhomogeneous Euclidean algebras or deformed 3d Poincaré algebras. The obtained algebras form a subclass in the known classification of quantum 3d inhomogeneous symmetry algebras [3] and can be compared with e.g. the recent results of [4], containing the list of deformed symmetry algebras that are associated with the Drinfel'd double structures (the latter are claimed to be a necessary ingredient in the quantization of 2+1d gravity).

References

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