The hyperbolic Einstein-Rosen bridge

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Using systematically isothermal coordinates we show that there exist three different maximal extensions of the original Einstein-Rosen bridge. One of them, the hyperbolic Einstein-Rosen bridge, has two-dimensional sections diffeomorphic to the covering space of an hyperboloid of revolution, a singularity satisfying the cosmic censorship and a bridge generated by light-like geodesics that can be traversed by time-like curves. The collapse process that might produce this object is an interesting open problem to be compared with the point-particle source obtained by Katanaev.

In this work, we proceed, as Kruskal and Szekeres did with the Schwarzschild metric, to introduce isothermal coordinates (U, V) in the original Einstein-Rosen bridge (ER) [1], expressed in coordinates (t, u), to eliminate its coordinate singularity at $u = 0$. The result is a smooth metric defined on a manifold with boundary: ERb $= \{(U, V) \in \mathrm{R}^2 \mid U^2 - V^2 \geq 0\}$, minus the point $(0, 0)$. This is a non maximal extension of the ER metric whose boundary corresponds to the curve $u = 0$ where ER was singular. The light-like geodesics of ERb are incomplete because of the boundary, therefore the bridge of ERb is not transitable. The most trivial way to obtain a maximal extension is to consider this space-time as a subspace of the Kruskal-Szekeres manifold [2], in which only spatial geodesics can traverse from one part to the other through the point $(0, 0)$. But there can exist more than one maximal extension of a metric, in the sense that all geodesics are complete [3]. In this case, one can find two other possible maximal extensions by a change topology, which can be obtained by gluing points of the boundary (a precedent of this method is the Ellpitic Kruskal-Szekeres space, obtained in a similar way [4]). These extensions eliminate the boundary, making them maximal. In this work, we have chosen one of them, denoted as hER, whose two-dimensional section $\theta, \phi = constant$ is diffeomorphic to the covering space of an hyperboloid of revolution, from which one has suppressed a point of its throat, that corresponds to the point $(0, 0)$ not contained in ERb. The metric imported from ERb on the hER manifold is smooth over the bridge, which is generated by light-like geodesics, and makes it traversable by time-like curves. However, a singularity, satisfying the cosmic censorship, appears on one of the two parts of this space-time joined by the bridge. The other possible extension by changing the topology corresponds, as pointed out by Poplawsky [5], to the one obtained by Guendelman et al. [6, 7]. The hER space-time differs from this one in three aspects: a) it has no source installed in the bridge, b) it has not closed time-like geodesics typical of wormholes with exotic sources, and c) it presents a singularity, though compatible with the cosmic censorship conjecture

Regarding the source of this object, it was proven by Katanaev [8, 9] that a point particle at infinite distance of the bridge is the source of the Einstein-Rosen metric. He interpreted this fact as repulsive gravity near the particle, but we look for a different interpretation. Following the procedure developed for the Schwarzschild metric [10], we propose a possible collapse process to produce the bridge. It consists of two phases: contraction in one side, and expansion in the other side, and the bridge would be created in the instant of maximum contracion. This kind of process would produce a part of the space-time, avoiding the singularity. We leave the in-depth analysis of this process for future work.

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