Embedding with Vaidya geometry

A. V. Nikolaev[1,*] and S. D. Maharaj[1]

[1] *Astrophysics and Cosmology Research Unit, School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa*

Locally a four-dimensional pseudo-Riemannian spacetime can be isometrically embedded in a higher dimensional Euclidean space for a spherically symmetric spacetime. The class one embedding equations are given by

$$\text{Gauss equation: } R^{\mu\nu}{}_{\gamma\delta} = \pm\left(b^{\mu}_{\gamma}b^{\nu}_{\delta} - b^{\mu}_{\delta}b^{\nu}_{\gamma}\right), \tag{1}$$

$$\text{Codazzi equation: } b^{\mu}_{\nu;\gamma} = b^{\mu}_{\gamma;\nu}, \tag{2}$$

where $R^{\mu\nu}{}_{\delta\gamma}$ is Riemann tensor, and Greek indices correspond to $[t, r, \phi, \theta]$. The generalized Vaidya metric is important in describing the exterior of a radiating star with a generalized atmosphere containing both a null dust and a null string fluid. The metric for the generalized Vaidya spacetime is

$$ds^2 = -\left(1 - \frac{2m(\nu, r)}{r}\right) d\nu^2 - 2d\nu dr + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{3}$$

with energy momentum tensor

$$T_{\mu\nu} = \varepsilon k_\mu k_\nu + (\rho + P)(k_\mu l_\nu + k_\nu l_\mu) + P g_{\mu\nu}, \tag{4}$$

where $l^\mu l_\mu = 0$ and $l^\mu k_\mu = -1$. In the above ρ is the null string energy density and P is the null string pressure. The matter tensor (4) describes an atmosphere which is a superposition of null dust (with ε), and null string fluid (with ρ and P).

It is therefore important to study the embedding properties of this geometry. We had shown that the class one embedding condition for the generalized Vaidya metric is a particular case of the Karmakar condition [1, 2]

$$R^{10}{}_{10} = \frac{R^{12}{}_{12}R^{20}{}_{20}}{R^{23}{}_{23}} - \frac{R^{12}{}_{20}R^{20}{}_{12}}{R^{23}{}_{23}}. \tag{5}$$

By solving the embedding conditions we get a subclass of the generalized Vaidya metric which is class one,

$$ds^2 = -\left(1 - C_2(\nu)(r + C_1(\nu))^2\right) d\nu^2 - 2d\nu dr + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{6}$$

Substituting into the Einstein field equations and solving, we obtain an equation of state for the corresponding generalized star atmosphere

$$P = \frac{1}{\kappa}\left(C \pm 2\sqrt{C(\rho\kappa + C)}\right).$$

[1] K. Karmarkar. *Proc. Ind. Acad. Sci. A*, 27(56), 1948.

[2] H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers, and E. Herlt. *Exact Solutions of Einstein's Field Equations.* Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2 edition, 2003.

* ilc@xhns.org