Bouncing cosmological solutions are found in a simple model of Einstein gravity coupled non-minimally to a self-interacting real scalar field. The characteristics of the solutions are presented and analyzed with an effective potential as a central tool. Bouncing solutions exist for the Higgs-like self interaction which is bounded from below, contrary to previous claims in favor of unbounded potentials. Appearance of higher power terms in the potential, gives rise to bouncing solutions even if the self-interaction potential has a symmetric minimum only. The bouncing solutions exist in a finite domain in parameter space with nonzero measure in the space of initial conditions. In other regions of parameter space, solutions with a Big Bang behavior exist, as well as others.