It is nowadays widely accepted that dark matter is an essential ingredient of the Universe. It is instrumental in explaining a wide range of astronomical phenomena: from enigmatic dynamics of galaxies and clusters to the large-scale structure of the Universe. Currently the most commonly accepted candidate is cold dark matter, which is successful in matching theoretical predictions on large cosmological scales. However, cold dark matter faces some difficulties on kpc to Mpc scales. One of the viable alternatives, which provides promising solutions to some of those challenges, is fuzzy dark matter, consisting of extremely light bosons with masses around 10^-23 eV. As shown by Khmelnitsky and Rubakov, such a field will produce oscillating gravitational potentials in nanohertz regime, which can be probed with pulsar timing arrays. We perform a search for evidence of fuzzy dark matter in the Galaxy using the latest Parkes Pulsar Timing Array 12-year data set. No statistically significant signal has been found and therefore we report upper limits on the local density of fuzzy dark matter. We also discuss the prospects of constraining fuzzy dark matter in the Square Kilometer era.