Some New Exact Solutions of Type D Admitting Maximal G_3 and G_4

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Abstract

Some new exact solutions of Einstein's field equations obtained in [M.Ziad (2018). Journal of Physics Communications 2 (11), 115011] will be discussed. These metrics are of type **D** and admit either a maximal Abelian G_3 or $G_4 \supset G_3$. There appears a class of metrics upto an arbitrary constant a and admits a maximal G_3. This class of metrics was missed in the investigations made in [Barnes, A. (1972). *Static perfect fluids in general relativity*, J. Phys. A: Math Gen. **5**, 374]. For $a > 0$, it appears to contain Kramer's type **D** metric [Kramer, D. (1988a). *Cylindrically symmetric static perfect fluids*, CQG **5**, 393] as a special case. It satisfies the equation of state $p = \omega$ with $\omega > 0$ for $a < 0$ and describes radiation, relativistic degenerate Fermi gas and probably very dense baryon matter according to the investigations made in [Zeldovich Y B and Novikov I D 1971. *Stars and relativity*, Relativistic Astrophysics vol 1 (Chicago, IL: University of Chicago Press)]. For a maximal G_4, the requirement for a perfect fluid reduces to a third order non-linear ODE. A complete solution of this ODE is achieved which gives three independent classes of perfect fluid spacetime metrics: a new class of metrics up to two arbitrary constants k_1 and k_2 which reduces to the empty solution of Levi Civita and Kasner for $k_1 = 0$, and to the perfect fluid solution found by Taub in implicit form [Taub, A. H. (1956). *Isentropic hydrodynamics in plane symmetric spacetimes,* Phys. Rev. **103**, 454], later found by Hojman and Santamarina [Hojman R. and Santamarina J. 1984, *Exact solutions of plane symmetric cosmological models*, JMP **25**, 1973], and by Collins [Collins C. B. 1985, *Static relativistic perfect fluids with spherical, plane, or hyperbolic symmetry*, JMP **26**, 2268] explicitly for $k_2 = 0$; another new class of metrics appears up to an arbitrary constant k and reduces to the Tabenski and Taub metric [Tabensky, R. R. and Taub, A. H. (1973). *Plane symmetric self gravitating fluids with pressure equal to energy density*, Commun. Math. Phys. **29**, 61] for $k < 0$, with $\omega < 0$. The other metric from this class gives $p = \omega$ with $(\omega > 0)$ for $k > 0$. Zeldovich and Novikov interpretation is also applicable on this metric while Tabenski and Taub [34] added that If in addition the motion of the gas is irrotational, then such a source has the same stress-energy tensor as that of the massless scalar field. Wainwright et. al. noted that the mass-less scalar field solutions can in turn be interpreted as vacuum solutions in the Brans-Dicke theory [Krasiński A1997. *Inhomogeneous Cosmological Models*, Cambridge:Cambridge University Press]; and a class of perfect fluid metrics up to two arbitrary functions $f(x)$ and $g(x)$, subject to a constraint is obtained which is capable of generating new exact solutions, three examples are constructed to ensure the existence of perfect fluid solutions from this class of metrics.