Spin and Center of mass comparison between the PN approach and the asymptotic formulation

C. N. Kozameh, J. I. Nieva, and G. D. Quiroga

I. ABSTRACT

In this work we analyse the similarities and differences between the equations of motion for the center of mass and intrinsic angular momentum for isolated sources of gravitational radiation obtained by two different formulations. One approach is based on the asymptotic formulation of the GR whereas the other relies on Post-Newtonian methods. Several conclusions are obtained which could be useful for further developments in both approaches.

The recent detections of gravitational waves made by LIGO [1, 2] have increased the interest in the study of binary systems and in the detection and characterization of the gravitational radiation emitted bythe binary coalescence. For these compact sources, it is very important to define the notion of center of mass and spin since the energy and momentum carried away by the gravitational wave induce a recoil to the center of mass of the coalesced system. Likewise, the spin of the resulting black hole or neutron star depends on the emitted gravitational wave.

To describe isolated sources the PN approximation relies on the definition of a point particle in Newtonian mechanics and its generalization to non trivial spacetimes. The gravitational radiation is computed in a given Bondi coordinate system. Matching conditions between the near zone and radiation zone allows to relate the source mass and current moments to the radiation fields[3].

The asymptotic formulation on the other hand, uses asymptotic flatness in general relativity to define global variables such as the momentum vector or the mass dipole/angular momentum 2-form of an isolated system. Some special congruences of cuts at null infinity are then associated with worldliness on a fiducial Minkowski. The center of mass worldline is then defined as the special congruence where the mass dipole term vanishes[4].

Since the definitions of center of mass and intrinsic angular momentum in both formulations are different, one does not expect to have similar equations of motion. However, before or after gravitational radiation is emitted, they should be able to yield the same measurable quantities of a given astrophysical system. For example, in binary coalescence, both formulations should give the same the position, velocity and spin of the final compact object assuming identical initial conditions[5]. Since the two formulations rely on completely different geometrical setups, it was an open question as to whether or not they should give identical measurable quantities and it was a motivation to find an answer.

In this work we show that the evolution equations for the global variables obtained in both formulations have some similarities and also some differences.

- Both formulations yield identical results for the flux-balance equations for the dynamic mass moment, total angular momentum and the Bondi 4-momemtum.[6, 7]

- The formulations differ in the dynamical evolutions for the center of mass and intrinsic angular momentum.[6, 7]

Regarding the time evolution of the intrinsic angular momentum we found that they differ by a non-vanishing term, even if we time average over a period of the gravitational wave and this difference is of the same order of magnitude of the remaining terms in equation (**??**).

The equations of motion for the center of mass also exhibit a difference between the two approaches. However, this difference might be zero or negligible for binary coalescence. If one computes this difference in newtonian mechanics for two point particles separated by a distance r in the adiabatic approximation and takes a time average over a period, this difference vanishes.

Since the differences are directly related to the definitions of center of mass and spin in both approaches, it is certainly worthwhile to work out different models to see whether or not the differences are significant or whether one can find a common setup for both approaches.

[1] LIGO Scientific Collaboration. Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, **116**, 061102, 2016.

[2] LIGO Scientific and Virgo Collaboration. Gw170104: Observation of a 50-solar-mass binary black hole coalescence at redshift 0.2. *Phys. Rev. Lett.*, **118**, 221101, 2017.

[3] L. Blanchet. Gravitational radiation from post-newtonian sources and inspiralling compact binaries. *Liv. Rev. Rel.*, **17**, 2, 2014.

[4] C. N. Kozameh and G. D. Quiroga. Center of mass and spin for isolated sources of gravitational radiation. *Phys. Rev. D*, **98**, 064050, 2016.

[5] L. Blanchet, M. S. Qusailah and C. M. Will. Gravitational recoil of inspiralling black-hole binaries to second post-newtonian order. *Class Quantum Grav.* **635**, *508-15*, 2005.

[6] L. Blanchet and G. Faye. Flux-balance equations for linear momentum and center-of-mass position of self-gravitating post-Newtonian systems. arXiv:1811.08966v2 [gr-qc] 18 Dec 2018.

[7] C. N. Kozameh, J. Nieva and G. D. Quiroga. Spin and Center of mass comparison between the PN approach and the asymptotic formulation *Phys. Rev. D*, **93**, 064005, 2018.