Round null surfaces in Kerr space-time

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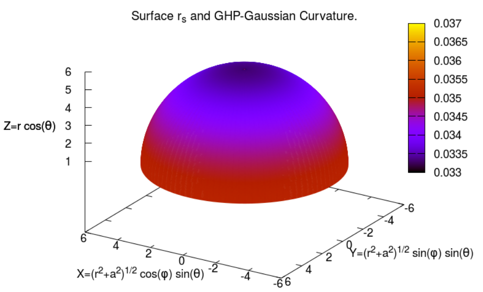
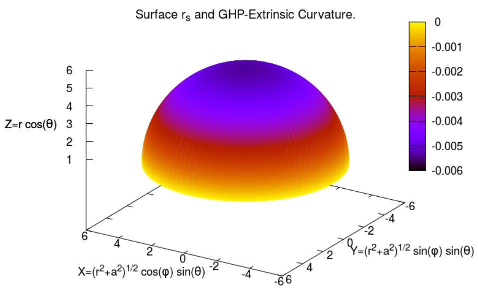
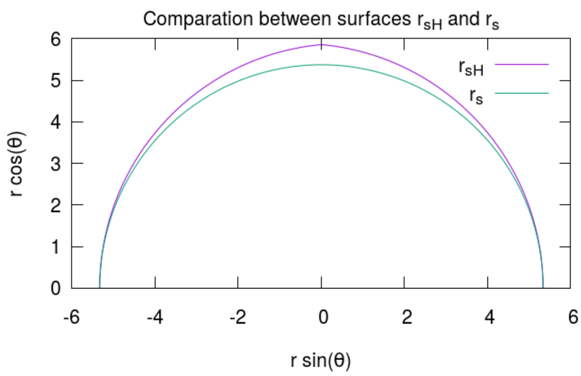
While the Kerr metric has deservedly been one of the most studied exact solutions, there appears to be a peculiar lack of natural null coordinates to describe a dual-null foliation of the space-time, meaning two families of null hypersurfaces intersecting in a two-parameter family of transverse spatial surfaces, such that the horizons are two of the hypersurfaces. We present a new definition for null coordinates, that we call **u** (out-going) and **v** (in-going), which are naturally adapted to the horizons. Our definition involves a differential equation which we solve numerically.

In our construction there naturally appear a family of spheres that are parameterized by r_s, which are the intersections of the null coordinates **u** and **v**. They can also be characterized in a coordinate independent way, by the intrinsic and extrinsic GHP curvature, given by $K_{Gaussian} = \bar{Q}_{GHP} + Q_{GHP}$ and $K_{Extrinsic} = i\left(\bar{Q}_{GHP} - Q_{GHP}\right)$, with $Q = \sigma\sigma' - \rho\rho' - \Psi_2$ given in terms of the spin coefficients of the GHP formalism. In the figure below, we show the smooth behavior of these curvatures through their numerical computation on a surface characterized by r_s, where (r, θ, ϕ) are in Boyer-Lindquist coordinates and a is the Kerr parameter.





Our work improves several attempts that can be found in the literature. A remarkable one is developed in [Hayward(2004)], where the null hypersurfaces they construct do not include the null geodesics along the axis of symmetry. This is due to the fact that their construction does not give a smooth hypersurface at the poles. In order to compare with ours coordinates, from [Hayward(2004)], we consider the null function $u* = t * -r*$. Where the analog to our natural spheres are the intersection of $u*$ with the Boyer-Lindquist coordinate t; that can be parameterized by r_{sH}. In the following graph one can be seen that for r_{sH} there is a discontinuity in the derivatives at $(\theta = 0)$, while for r_s it is clearly smooth.



Our approach is more related to the work in [Pretorius and Israel(1998)], whose treatment only covers the northern hemisphere, but also their expressions fail to deal with the north pole, and are very difficult to compute, even numerically.

Our new coordinates gives a new insight and are useful in the study of Kerr solution and the Kerr stability open problem. We plan to use them, in further works of Kerr perturbations.

References

[Hayward(2004)] S. A. Hayward, "Kerr black holes in horizon generating form", *Phys. Rev. Lett.* **92** (2004) 191101, arXiv:gr-qc/0401111.

[Pretorius and Israel(1998)] F. Pretorius and W. Israel, "Quasispherical light cones of the Kerr geometry", *Class. Quant. Grav.* **15** (1998) 2289–2301, arXiv:gr-qc/9803080.