Round null surfaces in Kerr space-time

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While the Kerr metric has deservedly been one of the most studied exact solutions, there appears to be a peculiar lack of natural null coordinates to describe a dual-null foliation of the space-time, meaning two families of null hypersurfaces intersecting in a two-parameter family of transverse spatial surfaces, such that the horizons are two of the hypersurfaces. We present a new definition for null coordinates, that we call $u$ (out-going) and $v$ (in-going), which are naturally adapted to the horizons. Our definition involves a differential equation which we solve numerically.

In our construction there naturally appear a family of spheres that are parameterized by $r_s$, which are the intersections of the null coordinates $u$ and $v$. They can also be characterized in a coordinate independent way, by the intrinsic and extrinsic GHP curvature, given by $K_{\text{Gaussian}} = Q_{\text{GHP}} + \bar{Q}_{\text{GHP}}$ and $K_{\text{Extrinsic}} = i (Q_{\text{GHP}} - \bar{Q}_{\text{GHP}})$, with $Q = \sigma' - \rho' - \Psi_2$ given in terms of the spin coefficients of the GHP formalism. In the figure below, we show the smooth behavior of these curvatures through their numerical computation on a surface characterized by $r_s$, where $(r, \theta, \phi)$ are in Boyer-Lindquist coordinates and $a$ is the Kerr parameter.

Our work improves several attempts that can be found in the literature. A remarkable one is developed in [Hayward(2004)], where the null hypersurfaces they construct do not include the null geodesics along the axis of symmetry. This is due to the fact that their construction does not give a smooth hypersurface at the poles. In order to compare with our coordinates, from [Hayward(2004)], we consider the null function $u^* = t^* - r^*$. Where the analog to our natural spheres are the intersection of $u^*$ with the Boyer-Lindquist coordinate $t$; that can be parameterized by $r_{sH}$. In the following graph one can be seen that for $r_{sH}$ there is a discontinuity in the derivatives at $(\theta = 0)$, while for $r_s$ it is clearly smooth.

Our approach is more related to the work in [Pretorius and Israel(1998)], whose treatment only covers the northern hemisphere, but also their expressions fail to deal with the north pole, and are very difficult tocompute, even numerically.

Our new coordinates gives a new insight and are useful in the study of Kerr solution and the Kerr stability open problem. We plan to use them, in further works of Kerr perturbations.

References