Spherical Thin-Shell Wormholes and Symmetry

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The concept of thin-shell wormhole (TSW) was introduced by Visser in 1989 [1, 2] in the hope of keeping the idea of wormholes alive by confining the exotic matter to a thin shell, called the throat of the TSW. Exotic matter, which inevitably emerges in the theories of wormholes, is an unwanted type of matter that violates the known energy conditions such as the weak energy condition (WEC) [3]. Pre-Visser's theories had the exotic matter distributed on the certain parts of the spacetime, if not all over it. However, Visser's so called cut-and-paste procedure allows us to confine such a notorious matter on a very limited part of the space, the TSW itself. Moreover, the cut-and-paste procedure has the advantage that can be applied to a vast variety of spacetimes [4–15], while before Visser only some certain spacetimes had the structure of a wormhole [16]. It is also worth mentioning that while TSWs are categorized as traversable wormholes not all the wormholes are considered to be traversable [17].

To construct a TSW by Visser's method in the spherical coordinates, consider two distinct Lorentzian spacetimes denoted by $(\Sigma, g)^{\pm}$. Out of each spacetime, a subset is cut such that no singularities or event horizons of any sort are included, i.e. $(\Upsilon, g)^{\pm} \subset (\Sigma, g)^{\pm}$ and $(\Upsilon, g)^{\pm} = \{x^{\mu}_{\pm} | r \geq a > r_e\}$, where r_e is any existed event horizon. Then, by pasting these two cuts at their common timelike hypersurface $\partial\Upsilon$, such that $\partial\Sigma \subset (\Sigma, g)^{\pm}$, one creates a complete Riemannian spacetime which provides a passage from one spacetime to the other. The hypersurface $\partial\Upsilon$ is indeed the throat of the TSW and contains the exotic matter. Note that, the coordinates of the two sides of the throat $x^{\mu}_{\pm}$, and more generally, the very nature of the two spacetimes does not necessarily be the same. Although most of the authors have been tending to consider same spacetime as the side spacetimes, recently such a mirror symmetry has been broken in some studies [18–20].

Suppose that the line element of the bulks are given by the static general spherically symmetric metrics

$$ds^2_{\pm} = -f_{\pm}(r_{\pm})\, dt^2_{\pm} + f^{-1}_{\pm}(r_{\pm})\, dr^2_{\pm} + h_{\pm}(r_{\pm})\, d\Omega^2_{\pm}, \tag{1}$$

where $f_{\pm}(r)$ and $h_{\pm}(r)$ are positive functions of the radial coordinates $r_{\pm}$, and $d\Omega_{\pm}$ are 2-spheres. are The line element on the hypersurface $\partial\Upsilon$ (the throat) is given by

$$ds^2_{\partial\Upsilon} = q^{\pm}_{ij} d\xi^i d\xi^j, \tag{2}$$

where ξ^i are the local coordinates on the shell and $q^{\pm}_{ij} = \frac{\partial x^{\mu}_{\pm}}{\partial \xi^i} \frac{\partial x^{\nu}_{\pm}}{\partial \xi^j} g^{\pm}_{\mu\nu}$ is the localized metric of $\partial\Upsilon$. The unit normals to the surface are also given by $n^{\pm}_{\mu} \frac{\partial x^{\mu}_{\pm}}{\partial \xi^i} = 0$; $n^{\pm}_{\mu} n^{\mu}_{\pm} = 1$. To count for the uniqueness of the TSW, $q^{-}_{ij} = q^{+}_{ij}$ must hold on the throat. In general relativity, this is called the first of Israel-Darmois junction conditions [22]. There also exists a second one. The second junction conditions impose a discontinuity on the extrinsic curvature tensor, given by

$$K^{\pm}_{ij} = -n^{\pm}_{\lambda} \left(\frac{\partial^2 x^{\lambda}_{\pm}}{\partial \xi^i \partial \xi^j} + \Gamma^{\lambda\pm}_{\alpha\beta} \frac{\partial x^{\alpha}_{\pm}}{\partial \xi^i} \frac{\partial x^{\beta}_{\pm}}{\partial \xi^j} \right), \tag{3}$$

where $\Gamma^{\lambda\pm}_{\alpha\beta}$ are the Christoffel symbols of the bulk spacetimes, compatible with $g^{\pm}_{\alpha\beta}$. By introducing $S^i_j = diag(-\sigma, p, p)$ as the stress-energy tensor of the perfect fluid on the throat, with σ and p being the energy density and lateral pressure, respectively, the second junction conditions admit

$$\left[K^i_j - \delta^i_j K\right]^+_- = -S^i_j, \tag{4}$$

where we symbolically have $[\Psi]^+_- = \Psi_+ - \Psi_-$ for a jump in quantity Ψ passing across the throat. Going through all the cumbersome calculations ends up with

$$\sigma = -\frac{h'}{h} \left[\sqrt{f_+ + \dot{a}^2} + \sqrt{f_- + \dot{a}^2} \right] \tag{5}$$

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and

$$p = \frac{\sqrt{f_+ + \dot{a}^2}}{2}\left[\frac{2\ddot{a} + f'_+}{f_+ + \dot{a}^2} + \frac{h'}{h}\right] + \frac{\sqrt{f_- + \dot{a}^2}}{2}\left[\frac{2\ddot{a} + f'_-}{f_- + \dot{a}^2} + \frac{h'}{h}\right]. \tag{6}$$

Herein, an overdot and a prime stand for a total derivative with respect to the proper time on the throat τ and the corresponding radial coordinates $r_\pm$, respectively. All the functions are measured at the location of the throat $r_\pm = a$. Also, note that due to the first junction condition $h_+ (a) = h_- (a) = h$, although this does not mean that $h'_+ (a) = h'_- (a) = h'$ necessarily holds. However, the conservation of energy, which is identified as

$$\sigma' + \frac{h'_\pm}{h} (\sigma + p) = 0, \tag{7}$$

must hold for both sides and therefore $h'_+ (a) = h'_- (a) = h'$.

In Eq. (), if $h' < 0$ then $\sigma > 0$; the WEC is satisfied and the matter is not exotic anymore. Nonetheless, the flare-out condition [27] implies $h' > 0$ which takes us back to square one. Although, generalizing the flare-out conditions, which are originally deduced for wormhole spacetimes, to TSWs is rather a matter of doubt [28]. Despite the existence of the exotic matter, there exist methods to either numerically (e.g. linear stability analysis) [21] or analytically (thermodynamic stability) [23] calculate for the stability of TSWs. The main question is that in what ways asymmetry of a TSW may contribute to its stability and, more fundamentally, may it provide us a way to be released from the exotic matter nightmare.

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