A Class of Integrable Metrics Coupled to Gauge Fields

Gabriel Luz Almeida
Graduate student at Universidade Federal de Pernambuco - Brazil
Supervisor: Carlos Batista

February 28, 2019

Abstract - The problem of integrability of the geodesic equation in general relativity arises naturally in the study of separability of the Hamilton-Jacobi equation for a particular Hamiltonian function called the geodesic Hamiltonian, defined on the cotangent bundle of spacetimes. It is well known from differential geometry that the existence of classes of coordinate systems that separates the Hamilton-Jacobi for this particular Hamiltonian function, the so-called separability structures, is intimately related to the existence of rank-2 Killing tensors, as well as commuting Killing vector fields, [1, 2]. In fact, this study leads to the most general form taken by the metric tensor in n dimensions containing a set of $m \leq n$ rank-2 Killing tensors in involution with each other (one of them always being taken to be the metric tensor itself) and $r = n - m$ commuting Killing vector fields, [2]. The close relationship between the notion of separability structures and the existence of sufficiently many symmetries to completely integrate the geodesic equations is manifest in this approach. In fact, the complete integrability of the geodesic equation will always hold for spaces admitting a separability structure. Based on this, in the present research project, starting with the most general four-dimensional spacetime possessing two commuting Killing vectors and a nontrivial Killing tensor, we analytically integrate Einstein-Yang-Mills equations for a completely arbitrary gauge group. It is assumed that the gauge field inherits the symmetries of the background and is aligned with the principal null directions of the spacetime. In particular, generalizations of the Kerr-NUT-(A)dS spacetime to contain non-abelian gauge fields as a source of matter and energy are presented, [3]. Besides this, when the coupling constant of the gauge fields is set to zero, the origin of the quadratic first integral in the Kerr spacetime found by Carter in [4] becomes better understood, [1, 5].

References

[1] S. Benenti and M. Francaviglia, *Remarks on certain separability structures and their applications to general relativity*, Gen. Relativ. Gravit. **10** (1979), 79.

[2] S. Benenti, *Separation of variables in the geodesic Hamilton-Jacobi equation*, Progr. Math **9** (1991), 1.

[3] G. L. Almeida and C. Batista, *Class of integrable metrics and gauge fields*, Phys. Rev. D **96** (2017), 084003.

[4] B. Carter, *Hamilton-Jacobi and Schrdinger separable solutions of Einstein's equations*, Comm. Math. Phys. **10** (1968), 280.

[5] A. Anabalón and C. Batista, *A class of integrable metrics*, Phys. Rev. D **93** (2016), 064079.