New gravitational lens equations for black holes with angular momentum

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There are many observations of black holes with angular momentum at Solar mass scales and supermassive ones, that have high intrinsic angular momentum[1]. It is of interest to study the effects of the existence of the angular momentum on the optical scalars associated to the gravitational lens produced by the black hole. Since the standard description of weak lenses, in the thin lens approximation, through the projected mass on the plane of the lens, does not describe black holes with angular momentum, several studies have dealt with the specific calculation of the Kerr geometry[2, 3, 4, 5, 6].

We present a new very simple exact expression for the Weyl curvature scalar Ψ_0; which is the component needed in any calculation involving the null *geodesic deviation equation*, of Kerr spacetime; namely:

$$\Psi_0 = -\frac{3M^{5/3}\mathbb{K}^2}{2\left(r - ia\cos(\theta)\right)^5}; \tag{1}$$

where M is the mass of the black hole, a the Kerr parameter, $\mathbb{K}$ a constant with spin-weight 1 which is related to the Carter's constant, and (r, θ) are Boyer-Lindquist coordinates.

We discuss how such expression allows for an efficient numerical calculation of weak lensing optical scalars in Kerr spacetime in an exact way without need to recur to the usual approach based on the notion of bending angle.

We also introduce a prescription for the observer frame which capture the notion of center of the black-hole which is very appropriated for the description of the observations. This construction is based on the unique null geodesic passing by the observer and belonging to the *center of mass null geodesic congruence*.

A numerical study and the results of the lensing effect is presented where the geometric values of the black hole are taken from the center supermassive black hole at M87; with mass of $M = 6.7 \times 10^9$ solar masses. The angular momentum of this black holes, according to [7], can be characterized by a Kerr parameter of $a = 0.98M$; which we have used in our calculations. Our treatment allows to consider any angle between the angular momentum and the line of sight of the observer.

References

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