

# Geometric flux formula for the gravitational Wilson loop

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## Abstract

Finding suitable diffeomorphism-invariant observables to probe gravity at the Planck scale is an essential part of addressing meaningful questions in quantum gravity. The non-infinitesimal Wilson loop of the four-dimensional Levi-Civita connection is a potentially interesting ingredient in the construction of such an observable. This has motivated our classical investigation of how and under what circumstances such Wilson loops can provide a (coarse-grained) measure of spacetime curvature. Invoking arguments similar to those used in the non-Abelian Stokes' theorem, we present an expression for a geometric flux, which in the presence of a totally geodesic surface relates non-infinitesimal gravitational Wilson loops to surface integrals of the scalar curvature.

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