

# The motion of localized sources in general relativity: gravitational self-force from quasilocal conservation laws

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An idealized “test” object in general relativity moves along a geodesic. However, if the object has a finite mass, this will create additional curvature in the spacetime, causing it to deviate from geodesic motion. If the mass is nonetheless sufficiently small, this effect can be treated perturbatively and is known as the gravitational self-force due to the object. The computation of the self-force is an open problem in gravitational physics today not only for basic foundational interest, but also for direct application in gravitational-wave astronomy. In particular, the observation of extreme-mass-ratio binaries by the next generation of space-based detectors such as LISA will rely crucially on an accurate modelling of the self-force driving their orbital evolution and gravitational wave emission. In this paper we present a novel derivation, based on conservation laws, of the basic equations of motion for this problem. They are formulated with the use of a quasilocal (rather than matter) stress-energy-momentum tensor—in particular, the Brown-York tensor—so as to capture gravitational effects in the object’s momentum flux, including the self-force. This approach offers a fresh geometrical picture from which to understand the self-force fundamentally, and potentially a useful new avenue for computing it practically.

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