Analytical and numerical treatment of perturbed black holes in horizon-penetrating coordinates

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Abstract

We investigate linear and non-linear scattering problems in black hole perturbation theory using our pseudospectral numerical relativity code 'bamps' [1–7]. As a first step towards quantifying the differences between the linear and nonlinear results, we provide an analytical approach to the linear problem but in horizon-penetrating coordinates, which are suited for numerical relativity. The response of the black hole to an initial configuration of a massless scalar field can be obtained from the Green's function for the problem, which is constructed from the linearly independent solutions of the confluent Heun equation [8] satisfying the necessary quasinormal mode boundary condition. The Green's function can then be used to compute the 'dynamic' excitation amplitude [9] of each quasinormal mode and the backscattering/tail contribution [10] for any observer outside the event horizon. The global solutions of the confluent Heun equation can also be used to construct a pure quasinormal mode which, in our coordinates is regular at horizon and can be evolved numerically, allowing for arbitrary long ringdown for observers placed on the event horizon. Making use of the hypergeometric solutions of the asymptotic confluent Heun equation, we also obtained simplified expressions for asymptotic observers. Numerical tests are then performed with specialized initial data and the presence of overtone modes are investigated and justified from the Green's function analysis. For fully nonlinear simulations, initial data is constructed using the conformal transverse traceless form of the constraints with the hyperbolic relaxation method [6] and evolved in the generalized harmonic formalism with the response of the black hole event horizon to the perturbation being studied by our event horizon finder. After a comparison of the linear and non-linear results, we discuss future work which involves handling 'perturbations' which are arbitrarily large.

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