The problem of constructing consistent Lorentz-CPT breaking extensions of various field theory models is actively discussed now. Within these studies, the problem of Lorentz-CPT symmetry breaking in gravity is of special importance. As it is well known, the main problems within this context are related with the fact that the group of general coordinate transformations playing the role of extension of the Lorentz group in a curved space-time, at the same time plays the role of the gauge group. As a result one faces the problem of breaking the gauge symmetry in a curved space-time. Another difficulty is related with the fact that while in the flat space the Lorentz symmetry breaking is introduced in terms of constant vectors (tensors), it is difficult to define such objects in a curved space-time.

All this implies that actually, there are three main directions in studying of Lorentz-breaking extensions of gravity: first, restricting to the case of weak (linearized) gravitational field where solving the problem of consistency of the Lorentz symmetry breaking with the gauge symmetry is easier, second, consideration of the Einstein gravity with the additive four-dimensional gravitational Chern-Simons term which breaks parity, and, for special choice of the Chern-Simons coefficient, breaks also the Lorentz symmetry, third, using the mechanism of spontaneous Lorentz symmetry breaking. Within our talk, we present the main results achieved by us within these approaches.

First, we consider the four-dimensional gravitational Chern-Simons term, with the main attention will be paid to the problem of its perturbative generation within different approaches [1, 2], including the finite temperature [3]. We demonstrate the finiteness and ambiguity of this term. Within this consideration, we also briefly discuss the possibility for perturbative generation of other Lorentz-breaking terms in linearized gravity.

Second, we discuss the bumblebee gravity model allowing for spontaneous Lorentz symmetry breaking due to introducing an additional vector field, and discuss the consistency of some simple metrics, especially the Gödel metric, within it [4].

Finally, we present some perspectives for studies of Lorentz-breaking extensions of gravity.