Abstract: Reference frames (or, more generally, systems) provide the vantage points from which to describe the remaining physics. Treating them fundamentally as quantum systems is inevitable in quantum gravity, where coordinates are a priori unavailable, but also in quantum foundations once accepting that all frames are physical systems. Both fields thus face the question of how to describe physics from the perspective of quantum reference systems and how the descriptions relative to different such choices are related. I will summarize a recent systematic method for such switches, which works in analogy to coordinate changes on a manifold, except that these 'quantum coordinate changes' proceed between different Hilbert spaces. This method relies on a new quantum reduction procedure, relating Dirac and reduced quantization, sets the stage for a quantum version of general covariance and applies to both temporal and spatial reference systems.