Total characteristics and the conformal Einstein field equations

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It is well know that the restriction of a symmetric hyperbolic system to one of its characteristic hypersurfaces gives rise to a set of (intrinsic) transport equations. Total characteristics are an extreme case of this: on one such hypersurface, the full symmetric hyperbolic evolution system reduces to transport equations. Total characteristics arise naturally in the study of the conformal structure of spacetimes —most notably, in the analysis of the Einstein equations in a neighbourhood of spatial infinity.

In [1] an initial value problem of the (conformal) Einstein field equations near spatial infinity has been introduced. This initial value problem has the properties that: the initial data and the equations are regular; and spatial and null infinity have a finite representation with their structure and location known *a priori*. In this representation spatial infinity is described in terms of a cylinder $\mathcal{I} \approx [-1, 1] \times \mathbb{S}^2$ which is a total characteristic of the conformal evolution equations. This property of the cylinder at spatial infinity allows to *transport* information from the initial hypersurface to null infinity and, in particular, enables the computation of asymptotic expansions. These expansions make it possible to relate in a precise manner the properties of the initial data to asymptotic behaviour —like the peeling of the Weyl tensor.

Similar totally characteristic structures naturally arise in the study of the conformal structure of: (a) asymptotically flat extremal black holes; (b) de Sitter-like black hole spacetimes.

In the case of asymptotically flat extremal black holes, the total characteristic arises at the *trumpet end* of the Cauchy hypersurfaces of the outer domain of communication and allows to transport information from the asymptotic end to the horizon. Thus, in particular, it allows to analyse the regularity of the horizon in terms of the properties of initial data. This construction, which was already known for the extremal Reissner-Nordström spacetime [2], can be implemented for a larger class of extremal (vacuum and electrovacuum) black holes. De Sitter-like black hole spacetimes also contain hypersurfaces with trumpet ends. In this case the trumpet ends are located at the *intersection* of the horizon with the (spacelike) conformal boundary and an analogous total characteristic hypersurface can be identified.

In both cases (a) and (b) it is possible to formulate a regular initial value problem for the conformal Einstein field equations in a neighbourhood of the total characteristic hypersurfaces which shares many of the structural properties of the problem at spatial infinity analysed by Friedrich. These initial value problems allow to study in a systematic manner the generic properties of asymptotic points in extremal and de Sitter-like black hole spacetimes.

References

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