

The future is not always open

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Abstract

Lorentzian geometry with non-smooth metrics (e.g. merely continuous, or Lipschitz, or C^{2-}) has received increasing interest in classical General Relativity during the last couple of years.

In this contribution we demonstrate the breakdown of several fundamentals of Lorentzian causality theory in low regularity. Most notably, chronological futures (defined naturally using locally Lipschitz curves) may be non-open, and may differ from the corresponding sets defined via piecewise C^1 -curves. By refining the notion of a causal bubble from (Chruściel and Grant, CQG 29, 2012) we characterize spacetimes for which such phenomena can occur, and also relate these to the possibility of deforming causal curves of positive length into timelike curves (push-up principle). The phenomena described here in a number of explicit examples are, in particular, relevant for recent synthetic approaches to low regularity Lorentzian geometry where, in the absence of a differentiable structure, causality has to be based on locally Lipschitz curves.

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