A perturbative approach to the construction of initial data on compact manifolds

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We discuss the implementation, to the case of compact manifolds, of the perturbative method of Friedrich-Butscher [1, 2] for the construction of solutions to the vacuum Einstein constraint equations. This method exploits the properties of the *extended constraint equations* —a larger system of equations whose solutions imply a solution to the Einstein constraints. In restricting to the case of closed initial hypersurfaces we bring to the foreground the more geometric aspects of the method, emphasising the key structural features of the extended constraints that enable such an approach. In short, the Friedrich–Butscher method may be divided into two stages:

(i) ***Construction of candidate solutions:*** derive a auxiliary system of equations, with elliptic linearisation, and apply the implicit function theorem to guarantee existence of solutions.

(ii) ***Sufficiency***: prove that the solutions to the auxiliary system constructed in *Step (i)* are also solutions to the extended constraint equations.

The method is applied to the construction of nonlinear perturbations of constant mean curvature initial data of constant negative sectional curvature. We prove the existence of a neighbourhood of solutions to the constraint equations around such initial data, with particular components of the extrinsic curvature and electric/magnetic parts of the spacetime Weyl curvature prescribed as free data. The space of such free data is parametrised explicitly.

The extended constraint equations can be seen as a particular case of the *conformal constraint equations* of Friedrich (see [3]), corresponding to a trivial conformal factor. The conformal constraint equations offer a promising alternative for the construction (on non-compact manifolds) of initial data with *controlled asymptotics*. A detailed understanding of the extended constraints is a necessary first step towards the study of the conformal constraint equations.

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References

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