Hamiltonian vs stability in alternative theories of gravity

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When a Hamiltonian density is bounded by below, we know that the lowest-energy state must be stable. One is often tempted to reverse the theorem and therefore believe that an unbounded Hamiltonian density always implies an instability. The main purpose of this talk is to pedagogically explain why this is erroneous. Stability is indeed a coordinate-independent property, whereas the Hamiltonian density does depend on the choice of coordinates. In alternative theories of gravity, like k-essence or Horndeski theories, the correct stability criterion is a subtler version of the well-known "Weak Energy Condition" of general relativity. As an illustration, this criterion is applied to an exact Schwarzschild-de Sitter solution of a beyond-Horndeski theory, which is found to be stable for a given range of its parameters, contrary to a claim in the literature.

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[1] E. Babichev, C. Charmousis, G. Esposito-Farèse and A. Lehébel, *Stability of Black Holes and the Speed of Gravitational Waves within Self-Tuning Cosmological Models*, Phys. Rev. Lett. **120** (2018) 241101 [arXiv:1712.04398 [gr-qc]].

[2] E. Babichev, C. Charmousis, G. Esposito-Farèse and A. Lehébel, *Hamiltonian unboundedness vs stability with an application to Horndeski theory*, Phys. Rev. D **98** (2018) 104050 [arXiv:1803.11444 [gr-qc]].