Neutron stars (NSs) are strongly self-gravitating objects. In a class of scalar-tensor gravity, nonperturbative effects take place in the strong field of isolated NSs (“spontaneous scalarization”) [1, 2], as well as in binary NSs (“induced scalarization” or “dynamical scalarization”) [3]. As a consequence, NSs are scalarized, and the strong equivalence principle, which is a key ingredient for Einstein’s general relativity [4, 5], is violated. This phenomenon is closely related to Landau’s phase transition theory where the energetically favoured branch is picked by the Nature [2, 6]. In an asymmetric binary system with NSs, we expect to observe dipole radiation [1, 2]. Observations with binary pulsars and gravitational waves show good agreement with the quadrupole formalism (and its post-Newtonian extension) in general relativity [7, 8]. Therefore, stringent constraints are cast on alternative gravity theories. In the talk, I will review the most recent observational constraints from binary pulsars and the binary NS GW170817 [9], and discuss the projected constraints that will come from the next generation of ground-based gravitational-wave detectors [7].