Spinorial Formalism and Symmetries in Six Dimensions

Carlos Batista*
Departamento de Física, Universidade Federal de Pernambuco, Recife, Pernambuco 50740-560, Brazil

Spinors are objects that carry the fundamental representation of the orthogonal gourp associated to a metric. Thus, in any space endowed with a metric one can introduce spinorial calculus as a useful geometrical tool. Besides its mathematical relevance, spinors are also of central importance to Physics, since fermionic degrees of freedom must be described in terms of spinors. In spite of these facts, spinorial calculus is still an underrated tool.

In four-dimensional general relativity, the works of R. Penrose and many others have proved that the spinorial formalism can be more powerful and natural than the usual tensor approach, as exemplified by the results obtained on the Petrov classification and on the fall-off behaviour of massless fields in asymptotically flat spacetimes [1]. To give a more recent example, spinors have been used to compute scattering amplitudes of gluons in Yang-Mills theories [2]. The breakthrough to attain such results was the use of the index spinorial formalism. Indeed, although it is definitely possible to attain several interesting results about spinors using the usual abstract notation and without fixing the dimension, it turns out that many practical manipulations of spinors depend strongly on the dimension. Since the index spinorial notation takes fully into account the dimensional specificities, its use generally lead to great benefits.

Given such motivations, the aim of this talk is to present the basic rules of the index spinorial formalism in six dimensions [3–7] and show how one can take advantage of these in order to integrate the Killing spinor equation in six dimensions [8]. Killing spinors are of primary relevance to supergravity theories, since compactifications in spaces possessing Killing spinors preserve part of the supersymmetry [9]. In addition, Killing spinors can be used to generate Killing vectors and Killing-Yano tensors [10], which lead to conserved charges that facilitate the integration of field equations in backgrounds possessing such spinors [11–15].

Keywords: Spinors; Killing Spinors; Killing-Yano tensors; Supergravity

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* carlosbatistas@df.ufpe.br