On the Integrability of Einstein's Equation and Killing Tensors

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It is well-known that some of the most important exact solutions of Einstein's equation, besides having some spacetime symmetries, mathematically represented by Killing vector fields, turn out to be endowed with the so-called hidden symmetries, which are mathematically represented by Killing tensors and Killing-Yano tensors [1]. Indeed, as an example, the n-dimensional Kerr-NUT-(A)dS class of solutions [2] is endowed with a Killing-Yano tensor of rank $n-2$ which, in its turn, generates a whole tower of conserved charges that allows full analytic integration of the geodesic motion [3] and some other field equations in this background [4–9]. From the mechanical point of view, Killing tensors and Killing-Yano tensors are symmetries of the phase space of the geodesic motion [10]. With such coincidence in mind, namely the fact that exact solutions frequently have non-trivial Killing tensors, it is natural to wonder about the converse. Does the assumption of the existence of hidden symmetries facilitates the integration of Einstein's equation? In this talk I will present some works that go along the latter path, investigating how the existence of hidden symmetries constrains the spacetime [11, 12], and how such constrained metric ansatz can be used to facilitate the process of integrating Einstein's equation [13]. In particular, some specific exact solutions found by using this procedure will be presented [14–16].

Keywords: Exact solutions; Hidden Symmetries; Integrability conditions; Gauge fields; Higher-dimensional general relativity

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